Algebra 2B
Exponent and Log Review

Name: Key

Date/Pd:____________________

EXPONENTS

OPERATION	RULES	EXAMPLES
Product (multiply *like* bases)	$a^m \bullet a^n = a^{m+n}$ *(Base does not change)*	$3^3 \bullet 3^2 = 3^5 = 243$ $n^5 \bullet n^7 = n^{12}$
Power of a Power	$\left(a^m\right)^n = a^{mn}$	$\left(2^3\right)^2 = 2^6 = 64$ $\left(m^4\right)^5 = m^{20}$
Power of a Product	$(ab)^m = a^m b^m$ *{do not confuse with (x+y)^n}*	$(4x)^3 = 64x^3$ $(x+4)^3 = (x+4)(x+4)(x+4)$ $(x+4)(x^2+8x+16)$ $\cancel{x^3}+\cancel{8x^2}+64+\cancel{4x^2}+\cancel{32x}+\cancel{16x}$ $\boxed{x^3+12x^2+48x+64}$
Quotient	$\dfrac{a^m}{a^n} = a^{m-n}\ ;\ a \neq 0$	$\dfrac{3^7}{3^3} = 3^4 = 81$ $\dfrac{3^4}{3^7} = \dfrac{1}{3^3} = \dfrac{1}{27}$
Power of a Quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}\ ;\ b \neq 0$	$\left(\dfrac{-3}{5}\right)^3 = \dfrac{-27}{125}$
Zero Exponent Property	$a^0 = 1$	$(2x+5)^0 = 1$
Negative Exponent Property	$a^{-m} = \dfrac{1}{a^m}\ ;\ \dfrac{1}{a^{-m}} = a^m\ ;\ a \neq 0$	$5^{-2} = \dfrac{1}{25}$ $\dfrac{1}{2x^{-3}} = \dfrac{x^3}{2}$

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LOGS

OPERATION	RULES	EXAMPLES
Rewrite	$\log_b a = n \leftrightarrow b^n = a$ $a > 0, b > 0, b \neq 1$	$\log_7 49 = 2 \leftrightarrow 7^2 = 49$
log = log (only 1 log per side)	$\log_b a = \log_b c$ then $a = c$	$\log_3 4 = \log_3 2^2 \leftrightarrow 4 = 2^2$
Multiplication = Addition	$\log_b ac = \log_b a + \log_b c$	$\log_5 7x = \log_5 7 + \log_5 x$
Division = Subtraction	$\log_b \frac{a}{c} = \log_b a - \log_b c$	$\log_9 \frac{8}{11} = \log_9 8 - \log_9 11$
Exponent = Multiplication	$n \log_b a = \log_b a^n$	$\log_3 5^7 = 7 \log_3 5$
Log = 0 **Ln = 0**	$\log_b 1 = 0$ $\ln 1 = 0$	$\log_{12} 1 = 0 \leftrightarrow 12^0 = 1$ $\ln 1 = 0 \leftrightarrow e^0 = 1$
Log = 1 **Ln = 1**	$\log_b b = 1$ $\ln e = 1$	$\log_7 7 = 1 \leftrightarrow 7^1 = 7$ $\ln e = 1 \leftrightarrow e^1 = e$
Change of Base Formula	$\log_b a = \frac{\log a}{\log b}$	$\log_8 15 = \frac{\log 15}{\log 8}$
Log as Exponent	$b^{\log_b a} = a$ $e^{\ln a} = a$	$6^{\log_6 11} = 11$ $e^{\ln 15} = 15$
Natural Log	$\ln x = \log_e x$	"ln" = "$\log_e$"

*** MUST always condense before you solve a log ***

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Simplify. (No Calculator)

1. $\frac{\cancel{14}^{7}x^5y}{\cancel{16}_{8}x^2y^3}$

$\frac{7x^{\cancel{5}\,3}\cancel{y}}{8\cancel{x}y^{\cancel{3}\,2}} = \boxed{\frac{7x^3}{8y^2}}$

2. $\left(\frac{3x^3y}{-6xy^3}\right)^2$

$\left(\frac{x^2}{-2y^2}\right)^2 \boxed{\frac{x^4}{4y^4}}$

3. $\frac{(m^2n)^{-1}}{m^2n^{-1}}$

$\frac{n}{m^2(m^2n)} = \boxed{\frac{n}{m^4n}}$

4. $\left(\frac{-4ab}{2c^3}\right)^{-2}\cdot\left(\frac{a^0}{c^{-1}}\right)^3$

$\left(\frac{-2ab}{c^3}\right)^{-2}\cdot(c)^3 = \frac{(c^3)^2}{(-2ab)^2}\cdot c^3$

$= \frac{c^6\cdot c^3}{4a^2b^2} = \boxed{\frac{c^9}{4a^2b^2}}$

5. $49^{\frac{1}{2}}$

$= \sqrt{49}$

$= \boxed{7}$

6. $\left(\frac{1000}{27}\right)^{-\frac{2}{3}}$

$\left(\frac{27}{1000}\right)^{\frac{2}{3}} = \frac{27^{\frac{2}{3}}}{1000^{\frac{2}{3}}} = \frac{3^2}{10^2} = \boxed{\frac{9}{100}}$

7. $\frac{(2x^{-3}y)^{-1}}{2y^{-3}}$

$\frac{y^3}{2(2x^{-3}y)} = \frac{y^3}{4x^{-3}y}$

$= \frac{y^3x^3}{4y} = \boxed{\frac{y^2x^3}{4}}$

8. $\log_{18}1+\log_7 7$

$0+1$

$\boxed{1}$

9. $\ln\sqrt[3]{e}$ $= \log_e e^{\frac{1}{3}}$

$= \boxed{\frac{1}{3}}$

10. $7^{\frac{1}{2}\log_7 9+2\log_7 5}$

$7^{\log_7 9^{\frac{1}{2}}+\log_7 25}$

$7^{\log_7 3+\log_7 25}$

$7^{\log_7 3\cdot 25} = \cancel{7^{\log_7}}75 = \boxed{75}$

11. $\log_{27}1$

$27^x=1$

$x=0$

12. $\log_8 64$

$8^x=64$

$x=2$

13. $\ln e^5$

$\log_e e^5$

$\boxed{5}$

14. $e^{3\ln 2}$

$e^{\log_e 2^3}$

$\cancel{e}^{\log_e 8}$

$\boxed{8}$

15. $\log_{64}4$

$(64)^x=4$

$(\cancel{2}^6)^x=\cancel{2}^2$

$6x=2$

$\boxed{x=\frac{1}{3}}$

Simplify. Write answers in exponential and radical form.

16. $\sqrt[5]{10}\cdot\sqrt[4]{1000}$

$10^{\frac{1}{5}}\cdot10^{\frac{3}{4}}$

$10^{\frac{1}{5}+\frac{3}{4}}$

$10^{\frac{4}{20}+\frac{15}{20}}$

$\boxed{10^{\frac{19}{20}}=\sqrt[20]{10^{19}}}$

17. $\sqrt[3]{32}\div\sqrt{8}\cdot\sqrt[4]{4}$

$\frac{32^{\frac{1}{3}}\cdot4^{\frac{1}{4}}}{8^{\frac{1}{2}}}$

$\frac{2^{\frac{5}{3}}\cdot2^{\frac{2}{4}}}{2^{\frac{3}{2}}}$ $\quad 2^{\frac{5}{3}+\frac{2}{4}-\frac{3}{2}}$

$= 2^{\frac{20}{12}+\frac{6}{12}-\frac{18}{12}}$

$2^{\frac{8}{12}}$ $\boxed{2^{\frac{2}{3}}=\sqrt[3]{4}}$

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Solve.

18. $\left[(x+1)^{\frac{3}{2}}\right]^{\frac{2}{3}} = (27)^{\frac{2}{3}}$

$x+1 = 27^{\frac{2}{3}}$

$x+1 = 3^2$

$x+1 = 9$

$\boxed{x = 8}$

19. $27^{x-2} = 9^{x+1}$

$\cancel{3}^{3(x-2)} = \cancel{3}^{2(x+1)}$

$3x-6 = 2x+2$

$\boxed{x = 8}$

20. $\left[(3x)^{-\frac{3}{2}}\right]^{-\frac{2}{3}} = (8)^{-\frac{2}{3}}$

$3x = 8^{-\frac{2}{3}}$

$3x = \frac{1}{8^{2/3}}$

$3x = \frac{1}{2^2}$

$\frac{3x}{3} = \frac{\frac{1}{4}}{3}$

$\boxed{x = \frac{1}{12}}$

21. $\left[(3x+1)^{-\frac{1}{2}}\right]^{-2} = \left(\frac{1}{4}\right)^{-2}$

$3x+1 = 4^2$

$3x+1 = 16$

$3x = 15$

$\boxed{x = 5}$

22. $\log_5(x+2) + \log_5(x+1) = \log_5 12$

$\cancel{\log_5}(x+2)(x+1) = \cancel{\log_5} 12$

$x^2+3x+2 = 12$

$x^2+3x-10 = 0$

$(x+5)(x-2) = 0$

$x=-5,\ x=2$

$\boxed{\{2\}}$

($x \neq -5$ since must have (+) value)

23. $\log_2(x^2-4) - \log_2(x+2) = 3$

$\log_2 \frac{x^2-4}{x+2} = 3$

$\frac{x^2-4}{x+2} = 2^3$

$\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}} = 8$

$x-2 = 8$

$x = 10$

$\boxed{\{10\}}$

24. $\log_{(x+1)} 8 = -\frac{3}{4}$

$\left[(x+1)^{-\frac{3}{4}}\right]^{-\frac{4}{3}} = (8)^{-\frac{4}{3}}$

$x+1 = 2^{-4}$

$x+1 = \frac{1}{16}$

$x = -\frac{15}{16}$

$\boxed{\{-15/16\}}$

25. $3\log_5(x+2) = 6$

$\log_5 (x+2)^3 = 6$

$\left[(x+2)^3\right]^{\frac{1}{3}} = (5^6)^{\frac{1}{3}}$

$x+2 = 5^2$

$x+2 = 25$

$x = 23$

$\boxed{\{23\}}$

26. Find $\log_{11} 3$. Round to the nearest thousandth.

$\frac{\log 3}{\log 11} \approx 0.458$