

ABSOLUTE VALUE EQUATIONS

Example 1: Solve a simple absolute value equation.

a. $|2x-1|=9$

$$\begin{array}{l} 2x-1=9 \\ +1 \quad +1 \\ \hline 2x=10 \\ \frac{2x}{2}=\frac{10}{2} \\ x=5 \end{array} \quad \begin{array}{l} 2x-1=-9 \\ +1 \quad +1 \\ \hline 2x=-8 \\ \frac{2x}{2}=\frac{-8}{2} \\ x=-4 \end{array}$$

$\{-4, 5\}$

b. $|5-x|=15$

$$\begin{array}{l} 5-x=15 \\ -5 \quad -5 \\ \hline -x=10 \\ \frac{-x}{-1}=\frac{10}{-1} \\ x=-10 \end{array} \quad \begin{array}{l} 5-x=-15 \\ -5 \quad -5 \\ \hline -x=-20 \\ \frac{-x}{-1}=\frac{-20}{-1} \\ x=20 \end{array}$$

$\{-10, 20\}$

Example 2: Rewrite an absolute value equation. Then solve.

a. $|x+3|-5=4$

$$\begin{array}{l} |x+3|-5=4 \\ +5 \quad +5 \\ \hline |x+3|=9 \end{array}$$

$$\begin{array}{l} x+3=9 \\ -3 \quad -3 \\ \hline x=6 \end{array} \quad \begin{array}{l} x+3=-9 \\ -3 \quad -3 \\ \hline x=-12 \end{array}$$

or $\{-12, 6\}$

① Isolate abs. value

② Split & solve

c. $\frac{1}{2}|2x-5|=3$

$$\begin{array}{l} \frac{1}{2}|2x-5|=3 \\ \cdot 2 \quad \cdot 2 \\ \hline |2x-5|=6 \end{array}$$

$$\begin{array}{l} 2x-5=6 \\ +5 \quad +5 \\ \hline 2x=11 \\ \frac{2x}{2}=\frac{11}{2} \\ x=\frac{11}{2} \end{array} \quad \begin{array}{l} 2x-5=-6 \\ +5 \quad +5 \\ \hline 2x=-1 \\ \frac{2x}{2}=\frac{-1}{2} \\ x=-\frac{1}{2} \end{array}$$

$\{-\frac{1}{2}, \frac{11}{2}\}$

**Must get absolute value alone 1st!*

b. $\frac{-2|x-1|}{-2}=\frac{-8}{-2}$

$$\begin{array}{l} \frac{-2|x-1|}{-2}=\frac{-8}{-2} \\ \hline |x-1|=4 \end{array}$$

**Never distribute into absolute value!*

$$\begin{array}{l} x-1=4 \\ +1 \quad +1 \\ \hline x=5 \end{array} \quad \begin{array}{l} x-1=-4 \\ +1 \quad +1 \\ \hline x=-3 \end{array}$$

$\{-3, 5\}$

d. $\frac{2|3-x|}{2}=\frac{-8}{2}$

$$\begin{array}{l} \frac{2|3-x|}{2}=\frac{-8}{2} \\ \hline |3-x|=-4 \end{array}$$

No Solution

**Absolute value cannot equal a negative!*

$$e. \frac{|3x+5|}{2} - 6 = 2$$

$$\begin{array}{r} +6 +6 \\ \hline \end{array}$$

$$\{ -7, \frac{11}{3} \}$$

$$g. \frac{|x|}{3} - 2 = -1$$

$$\begin{array}{r} +2 +2 \\ \hline \end{array}$$

$$2. \frac{|3x+5|}{2} = 8 \cdot 2$$

$$3. \frac{|x|}{3} = 1 \cdot 3$$

$$|3x+5| = 16$$

$$\begin{array}{r} 3x+5=16 \\ -5 \quad -5 \\ \hline 3x=11 \\ \frac{3x}{3} = \frac{11}{3} \quad x = \frac{11}{3} \end{array}$$

$$\begin{array}{r} 3x+5=-16 \\ -5 \quad -5 \\ \hline 3x=-21 \\ \frac{3x}{3} = \frac{-21}{3} \quad x = -7 \end{array}$$

$$|x| = 3$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x = 3 \quad x = -3 \end{array}$$

$$\{ -3, 3 \}$$

$$f. |-2x+3| + 2 = 5$$

$$\begin{array}{r} -2 \quad -2 \\ \hline \end{array}$$

$$h. 3|2x-7| - 5 = 4$$

$$\begin{array}{r} +5 +5 \\ \hline \end{array}$$

$$|-2x+3| = 3$$

$$\begin{array}{r} -2x+3=3 \\ -3 \quad -3 \\ \hline -2x=0 \\ \frac{-2x}{-2} = \frac{0}{-2} \\ x = 0 \end{array}$$

$$\begin{array}{r} -2x+3=-3 \\ -3 \quad -3 \\ \hline -2x=-6 \\ \frac{-2x}{-2} = \frac{-6}{-2} \\ x = 3 \end{array}$$

Careful!
Don't distribute!
Divide each side by 3

$$3|2x-7| = 9$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$|2x-7| = 3$$

$$\begin{array}{r} 2x-7=3 \\ +7 \quad +7 \\ \hline 2x=10 \\ \frac{2x}{2} = \frac{10}{2} \\ x = 5 \end{array}$$

$$\begin{array}{r} 2x-7=-3 \\ +7 \quad +7 \\ \hline 2x=4 \\ \frac{2x}{2} = \frac{4}{2} \\ x = 2 \end{array}$$

$$\{ 0, 3 \}$$

$$\{ 2, 5 \}$$

YOU TRY!

$$1. |x+3| + 2 = 8$$

$$\begin{array}{r} -2 \quad -2 \\ \hline \end{array}$$

$$2. |4x+2| + 6 = 4$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$|x+3| = 6$$

$$|4x+2| = -2$$

$$\begin{array}{r} x+3=6 \\ -3 \quad -3 \\ \hline x=3 \end{array}$$

$$\begin{array}{r} x+3=-6 \\ -3 \quad -3 \\ \hline x=-9 \end{array}$$

No Solution

$$\{ -9, 3 \}$$

$$3. \frac{1}{2}|x| - 3 = 6$$

$$2. \frac{1}{2}|x| = 9 \cdot 2$$

$$|x| = 18$$

$$x = 18 \quad x = -18$$

$$\{-18, 18\}$$

$$4. 3|3x-6| - 1 = 8$$

$$\frac{3|3x-6|}{3} = \frac{9}{3}$$

$$|3x-6| = 3$$

$$3x-6 = 3$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

$$3x-6 = -3$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$x = 1$$

$$\{1, 3\}$$

$$5. 2|2x-5| - 1 = 9$$

$$\frac{2|2x-5|}{2} = \frac{10}{2}$$

$$|2x-5| = 5$$

$$2x-5 = 5$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

$$2x-5 = -5$$

$$\frac{2x}{2} = \frac{0}{2}$$

$$x = 0$$

$$\{0, 5\}$$

$$6. \frac{|x|}{2} + 5 = 3$$

$$2. \frac{|x|}{2} = -2 \cdot 2$$

$$|x| = -4$$

No Solution!

*Absolute value cannot equal a neg!

