

Ratio: Comparison of two quantities by division

The ratio of a to b can be written: $a:b$, $\frac{a}{b}$, a to b $\{b \neq 0\}$

Proportion: Statement that two ratios are equal

A ratio uses division to compare two quantities.

Example 1a: CR South Football plays 7 games at home and 4 games away.

Write the ratio in three different ways:

a) home games to away games

home : away
 $7:4$, $\frac{7}{4}$, 7 to 4

b) home games to all games

home : all
 $7:11$, $\frac{7}{11}$, 7 to 11
 $7 + 4 = 11$ ← all games

Example 1b: At a carwash fundraiser, 18 ninth grade students and 14 tenth grade students worked the first shift.

Write the ratio in three different ways:

a) 9th graders to 10th graders

9th : 10th
*reduce! $18:14$, $\frac{18}{14}$, 18 to 14
 $9:7$, $\frac{9}{7}$, 9 to 7

b) 9th graders to all students

9th : all
 $18:32$, $\frac{18}{32}$, 18 to 32
 $9:16$, $\frac{9}{16}$, 9 to 16
 $all: 18+14=32$
*Reduce!

Try it!

Derek and his brother decide to combine their CD collections. Derek has 44 CDs, and his brother has 52 CDs.

Write the ratio in three different ways:

1. Derek's CDs to his brother's

Derek : brother
 $11:13$, $\frac{11}{13}$, 11 to 13
 $44 + 52 = 96$ ← total

2. Derek's CDs to total

Derek : total
 $11:24$, $\frac{11}{24}$, 11 to 24

3. In a class bake sale, there were 50 brownies sold and 44 cookies sold. Write the ratio in three different ways:

a) brownies to total items $\frac{50}{94}$

$\frac{25}{47}$, $25:47$, 25 to 47

b) cookies to brownies $\frac{44}{50}$ *Watch order!

$\frac{22}{25}$, 22 to 25 , $22:25$

Rate: ratio of two quantities with different units

Unit Rate: rate with 2nd quantity of 1 unit

ex: miles/hour ← per 1 hour miles/gallon ← 1 gallon

Example 2a: Cory earns \$52.50 in 7 hours. Find the unit rate in dollars per hour.

$$\frac{\text{dollars}}{\text{hours}} \quad \frac{52.50}{7} = \boxed{\$7.50/\text{hour}}$$

Example 2b: If a car travels 45 miles in 30 min, what is the rate at which the car is travelling in miles per hour?

min → hour
30 → $\frac{30}{60} = \frac{1}{2}$ hr

*Change minutes to hours

$$\frac{\text{miles}}{\text{hour}} \quad \frac{45}{0.5} = \boxed{90 \text{ miles/hour}}$$

Try it!

1. Ralf Laue of Germany flipped a pancake 416 times in 120 seconds to set the world record. Find the unit rate in flips per second. Round to the nearest hundredth.

$$\frac{\text{flips}}{\text{second}} \quad \frac{416}{120} = \boxed{3.47 \text{ flips/second}}$$

tenth hundredth

*Need to Round!
0 1 2 3 4 stays the same!
5 6 7 8 9 Round up!

In the proportion $\frac{a}{b} = \frac{c}{d}$ the products of $a \times d = b \times c$ is called the cross products property.

*Cross multiply over =

Cross Products Property

WORDS	NUMBERS	ALGEBRA
In a proportion, cross products are equal.	$\frac{2}{3} = \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$	If $\frac{a}{b} = \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$ then $ad = bc$.

Example 3: Solve each proportion by using the cross products property.

a.) $\frac{1}{3} = \frac{x}{9}$

$$3x = 1(9)$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$\boxed{x = 3}$$

b.) $\frac{-1}{5} = \frac{3}{2x}$

$$-1(2x) = 5(3)$$

$$\frac{-2x}{-2} = \frac{15}{-2}$$

$$\boxed{x = \frac{-15}{2}}$$

c.) $\frac{-5}{2} = \frac{x}{8}$

$$-5(8) = 2(x)$$

$$\frac{-40}{2} = \frac{2x}{2}$$

$$\boxed{-20 = x}$$

$$d.) \frac{6}{y-3} = \frac{2}{7}$$

$$6(7) = 2(y-3)$$

$$42 = 2y - 6$$

$$\begin{array}{r} +6 \\ \hline 48 = 2y \end{array}$$

$$\frac{48}{2} = \frac{2y}{2}$$

$$\boxed{24 = y}$$

$$e.) \frac{g+3}{5} = \frac{7}{4}$$

$$5(7) = 4(g+3)$$

$$35 = 4g + 12$$

$$\begin{array}{r} -12 \\ \hline 23 = 4g \end{array}$$

$$\frac{23}{4} = \frac{4g}{4}$$

$$\boxed{\frac{23}{4} = g}$$

$$f.) \frac{6}{5} = \frac{x-4}{10}$$

$$6(10) = 5(x-4)$$

$$60 = 5x - 20$$

$$\begin{array}{r} +20 \\ \hline 80 = 5x \end{array}$$

$$\frac{80}{5} = \frac{5x}{5}$$

$$\boxed{16 = x}$$

$$g.) \frac{1}{y} = \frac{1}{6y-1}$$

$$y(1) = 1(6y-1)$$

$$y = 6y - 1$$

$$\begin{array}{r} -6y \\ \hline -5y = -1 \end{array}$$

$$\frac{-5y}{-5} = \frac{-1}{-5}$$

$$\boxed{y = \frac{1}{5}}$$

$$h.) \frac{3x+5}{14} = \frac{x}{3}$$

$$3(3x+5) = 14x$$

$$9x + 15 = 14x$$

$$\begin{array}{r} -9x \\ \hline 15 = 5x \end{array}$$

$$\frac{15}{5} = \frac{5x}{5}$$

$$\boxed{3 = x}$$

$$i.) \frac{18}{14} = \frac{x+2}{x}$$

$$18x = 14(x+2)$$

$$18x = 14x + 28$$

$$\begin{array}{r} -14x \\ \hline 4x = 28 \end{array}$$

$$\frac{4x}{4} = \frac{28}{4}$$

$$\boxed{x = 7}$$

$$j.) \frac{1}{x-3} = \frac{3}{x-5}$$

$$1(x-5) = 3(x-3)$$

$$x - 5 = 3x - 9$$

$$\begin{array}{r} -x \\ \hline -5 = 2x - 9 \end{array}$$

$$\begin{array}{r} +9 \\ \hline 4 = 2x \end{array}$$

$$\frac{4}{2} = \frac{2x}{2}$$

$$\boxed{x = 2}$$

$$k.) \frac{x-1}{3} = \frac{x+1}{5}$$

$$5(x-1) = 3(x+1)$$

$$5x - 5 = 3x + 3$$

$$\begin{array}{r} -3x \\ \hline 2x - 5 = 3 \end{array}$$

$$\begin{array}{r} +5 \\ \hline 2x = 8 \end{array}$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$\boxed{x = 4}$$

$$l.) \frac{m}{5} = \frac{m-6}{4}$$

$$4m = 5(m-6)$$

$$4m = 5m - 30$$

$$\begin{array}{r} -5m \\ \hline -m = -30 \end{array}$$

$$\frac{-m}{-1} = \frac{-30}{-1}$$

$$\boxed{m = 30}$$

Example 4: Solve word problems using proportions.

- a. You are making fresh salsa! Usually, you use 30 tomatoes and make 12 pints of salsa. This time, you only want to make a little bit of salsa...4 pints. How many tomatoes should you use??

tomatoes
pints

$$\frac{30}{12} = \frac{x}{4}$$

$$12x = 120$$

$$\frac{12x}{12} = \frac{120}{12}$$

$$\boxed{x = 10}$$

10 tomatoes

- b. The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?

faculty
students

$$\frac{1}{15} = \frac{x}{675}$$

$$\frac{15x}{15} = \frac{675}{15}$$

$$x = 45$$

$x = \#$ of faculty members

45 faculty members

Try it! Solve word problems using proportions.

1. The ratio of games won to games lost for a baseball team is 3:2. The team won 18 games. How many games did the team lose?

won
lost

$$\frac{3}{2} = \frac{18}{x}$$

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

$x =$ games lost

12 games lost

2. You are waiting to buy concert tickets. Every 10 minutes, the cashier helps 3 people. There are 11 people in the line in front of you. Write and solve a proportion that can determine how long you have to wait in line.

minutes
people

$$\frac{10}{3} = \frac{x}{11}$$

$$\frac{3x}{3} = \frac{110}{3}$$

$$x = 36.66$$

you have to wait about 37 min

Example 5: Write and solve the proportion

- a) 3 is to 8 as x is to 32.

$$\frac{3}{8} = \frac{x}{32}$$

$$\frac{8x}{8} = \frac{96}{8}$$

$$x = 12$$

- b) 12 is to 18 as d is to 27.

$$\frac{12}{18} = \frac{d}{27}$$

$$12(27) = 18d$$

$$324 = 18d$$

$$\frac{324}{18} = \frac{18d}{18}$$

$$18 = d$$

Try it!

1. $\frac{y}{25} = \frac{3}{5}$

$$\frac{5y}{5} = \frac{75}{5}$$

$$y = 15$$

2. x is to 21 as 40 is to 28.

$$\frac{x}{21} = \frac{40}{28}$$

$$\frac{28x}{28} = \frac{840}{28}$$

$$x = 30$$