

rate of change is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$\Delta = \text{change}$

Rate of change = $\frac{\Delta y}{\Delta x}$ OR Rate of change = $\frac{\text{change in } y}{\text{change in } x}$

Example 1: The table shows the average temperature (°F) for five months in a certain city. Find the rate of change for each time period. During which time period did the temperature increase at the fastest rate?

| Month | 2 | 3 | 5 | 7 | 8 |
|------------|----|----|----|----|----|
| Temp. (°F) | 56 | 56 | 63 | 71 | 72 |

Dependent variable (y): temperature

Independent variable (x): # of months

Rate of change from month 2 to 3: $\frac{56-56}{3-2} = \frac{0}{1} = 0^\circ \text{F/month}$

Rate of change from month 3 to 5: $\frac{63-56}{5-3} = \frac{7}{2} = 3.5^\circ \text{F/month}$

Rate of change from month 5 to 7: $\frac{71-63}{7-5} = \frac{8}{2} = 4^\circ \text{F/month}$

Rate of change from month 7 to 8: $\frac{72-71}{8-7} = \frac{1}{1} = 1^\circ \text{F/month}$

The temperature increase increased the fastest from month 5-7

Example 2: The table shows the balance of a bank account on different days of the month. Find the rate of change during each time interval. During which time interval did the balance decrease at the greatest rate?

| Day | 1 | 6 | 16 | 22 | 30 |
|--------------|-----|-----|-----|-----|-----|
| Balance (\$) | 550 | 285 | 210 | 210 | 175 |

Dependent variable: balance (\$)

Independent variable: # of days

Rate of change from day 1 to 6: $\frac{285-550}{6-1} = \frac{-265}{5} = \$-53/\text{day}$

Rate of change from day 6 to 16: $\frac{210-285}{16-6} = \frac{-75}{10} = \$-7.50/\text{day}$

Rate of change from day 16 to 22: $\frac{210-210}{22-16} = \frac{0}{6} = \$0/\text{day}$

Rate of change from day 22 to 30: $\frac{175-210}{30-22} = \frac{-35}{8} = \$-4.38/\text{day}$

The balance decreased at the greatest rate from Day 1 to Day 6

Slope of a Line

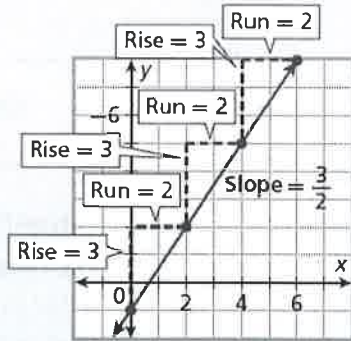
The **rise** is the difference in the y -values of two points on a line.

The **run** is the difference in the x -values of two points on a line.

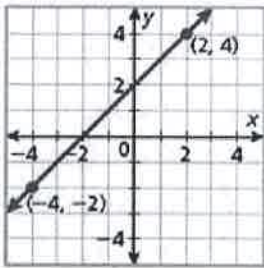
The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

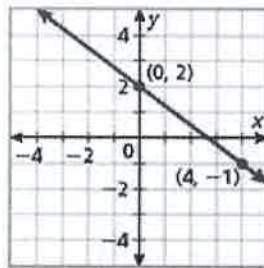
(Remember that y is the dependent variable and x is the independent variable.)



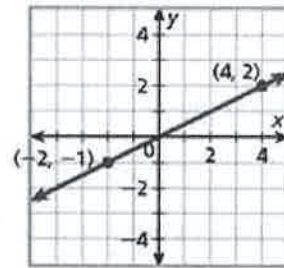
Example 3: Find the slope of each line.



a.)



b.)



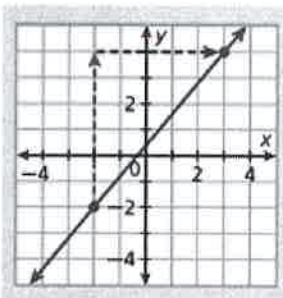
c.)

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{6} = \boxed{1}$$

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{4} = \boxed{-\frac{3}{4}}$$

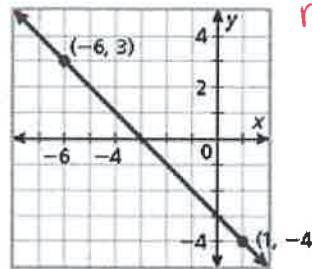
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

Try it! Find the slope of each line.



1.

positive



2.

negative

$$\text{Slope} = \frac{6}{5} = \boxed{\frac{6}{5}}$$

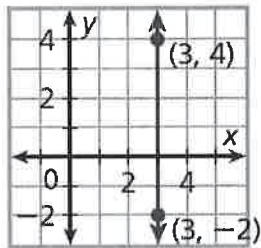
$$\text{Slope} = \frac{-7}{7} = \boxed{-1}$$

Special Cases: Vertical and Horizontal lines

Example 4: Find the slope of each line.

a.) The slope of a vertical line is:

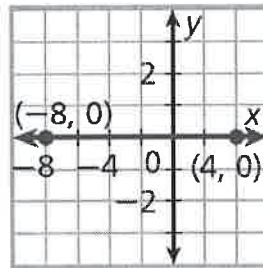
b.) The slope of a horizontal line is:



Rise: 6

Run: 0

Slope: $\frac{6}{0} = \text{Undefined}$



Rise: 0

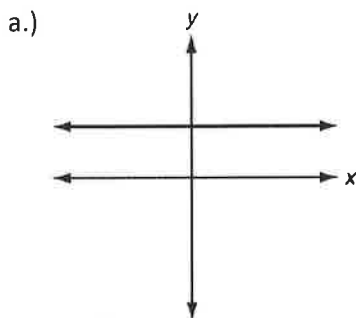
Run: 6

Slope: $\frac{0}{6} = 0$

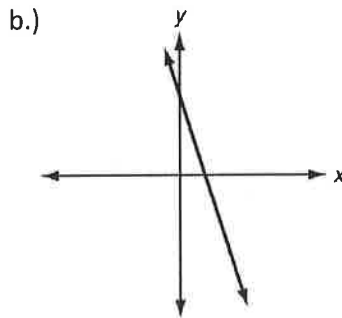
Example 5: Tell whether the slope of each line is positive, negative, zero, or undefined.

| Positive Slope | Negative Slope | Zero Slope | Undefined Slope |
|--------------------------------|--------------------------------|-----------------|-----------------|
| | | | |
| Line rises from left to right. | Line falls from left to right. | Horizontal line | Vertical line |

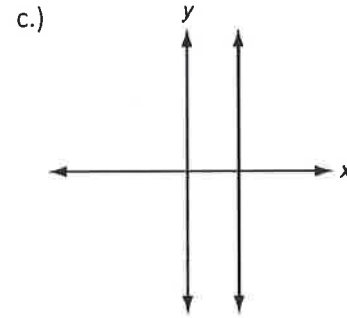
Try it! Tell whether the slope of each line is positive, negative, zero, or undefined.



Zero



Negative



Undefined

HOY

- *horizontal line
- *zero slope
- * $y = _$

VUX

- *vertical line
- *undefined slope
- * $x = _$

Example 6: The slope of a line is $\frac{1}{3}$ and the rise is 6. What is the run? $x = \text{run}$

$$m = \frac{\text{rise}}{\text{run}}$$

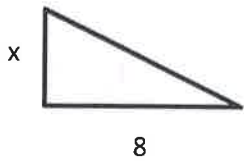
$$\frac{1}{3} = \frac{6}{x}$$

$$x = 18$$

The run is 18

Example 7: Given the slope, find the value of x using $\frac{\text{rise}}{\text{run}}$.

a.) Slope = $-\frac{1}{2}$



rise is -4

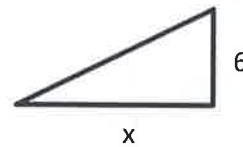
$$m = \frac{\text{rise}}{\text{run}}$$

$$-\frac{1}{2} = \frac{x}{8}$$

$$-\frac{8}{2} = \frac{2x}{2}$$

$$-4 = x$$

b.) Slope = $\frac{2}{3}$



run is 9

$$m = \frac{\text{rise}}{\text{run}}$$

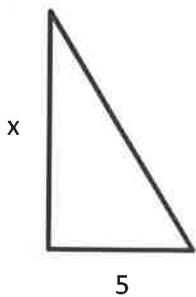
$$\frac{2}{3} = \frac{6}{x}$$

$$\frac{2x}{2} = \frac{18}{2}$$

$$x = 9$$

Try it!

1. Slope = $-\frac{5}{2}$

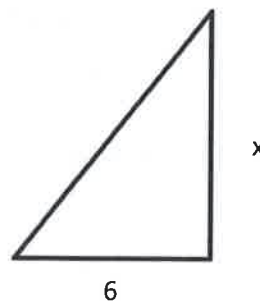


$$-\frac{5}{2} = \frac{x}{5}$$

$$\frac{2x}{2} = \frac{-25}{2}$$

$$x = -12.5$$

2. Slope = $\frac{8}{3}$



$$\frac{8}{3} = \frac{x}{6}$$

$$\frac{3x}{3} = \frac{48}{3}$$

$$x = 16$$

3. The slope of a line is $\frac{1}{2}$ and the run is 7. What is the rise?

$$\frac{\text{rise}}{\text{run}}$$

$$\frac{1}{2} = \frac{x}{7}$$

$$\frac{2x}{2} = \frac{7}{2}$$

$$x = 3.5$$