Algebra 1
Mrs. Moore/Ms. Paprocki

Name: Key

5.4 Notes – Solving Special Systems

When two lines intersect at a point, there is exactly one solution to the system. Systems with at least one solution are called **consistent.**

When the two lines in a system do not intersect they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is **inconsistent.**

Systems with NO solution-Lines are parallel

Ex 1: Show that the given system has no solution.

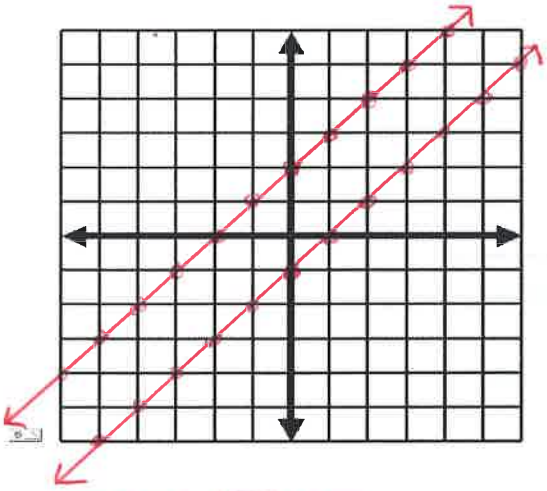
$$y = x - 1$$
$$-x + y = 2$$

Method 1: Compare slopes and y intercept

$y = x - 1$ $\quad$ $y = x + 2$

$m = 1$ $\quad$ $m = 1$

$b = -1$ $\quad$ $b = 2$



No Solution
Inconsistent

Method 2: Solve algebraically

Substitution:

$y = x - 1$

$-x + y = 2$

$-x + x - 1 = 2$

$-1 = 2$

False → No Solution $\varnothing$

Elimination:

$y = x - 1$ → $y = x - 1$

$-x + y = 2$ $\quad$ $-x \quad -x$

$-x + y = -1$

$-1(-x + y = 2)$

$-x + y = -1$

$x - y = -2$

$-x + y = -1$

$0 = -3$

False

No Solution $\varnothing$

Systems with INFINITELY MANY Solutions-Lines are coinciding (same exact line)

Ex 2: Show that the given system has infinitely many solutions.

$$y = 2x + 1$$
$$2x - y + 1 = 0$$

Method 1: Compare slopes and y intercept

$y = 2x + 1$

$m = 2$

$b = 1$

$2x - y + 1 = 0$

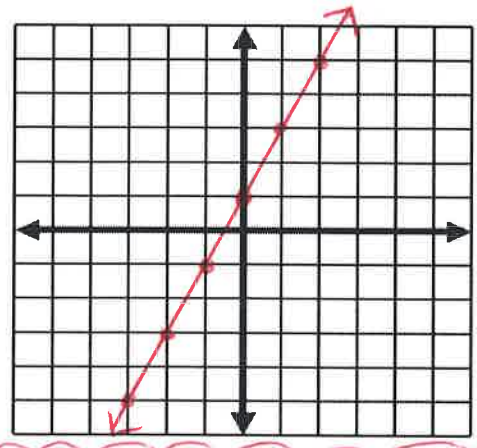
$+y \quad +y$

$2x + 1 = y$

$y = 2x + 1$

$m = 2$

$b = 1$



Infinitely many solutions → consistent dependent

Method 2: Solve algebraically

Substitution:

$y = 2x + 1$

$2x - y + 1 = 0$

$2x - (2x + 1) + 1 = 0$

$2x - 2x - 1 + 1 = 0$

$0 = 0$

TRUE → Infinitely Many Solutions

Elimination:

$y = 2x + 1$

$-2x \quad -2x$

$-2x + y = 1$

$2x - y + 1 = 0$

$-1 \quad -1$

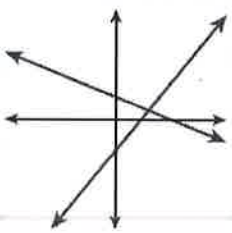
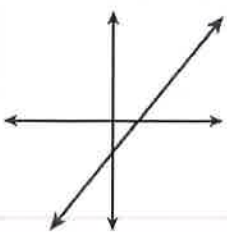
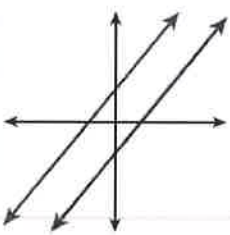
$2x - y = -1$

$-2x + y = 1$

$0 = 0$

TRUE → Infinitely many solutions

Classification of Systems of Linear Equations

CLASSIFICATION	CONSISTENT AND INDEPENDENT	CONSISTENT AND DEPENDENT	INCONSISTENT
Number of Solutions	Exactly one	Infinitely many	None
Description	Different slopes	Same slope, same *y*-intercept	Same slope, different *y*-intercepts
Graph	Intersecting lines 	Coincident lines 	Parallel lines 

Classify each system and give the number of solutions.

1.) $2y = x + 2$
$-\frac{1}{2}x + y = 1$

$\frac{2y}{2} = \frac{x}{2} + \frac{2}{2}$

$y = \frac{1}{2}x + 1$

$-\frac{1}{2}x + y = 1$

$+\frac{1}{2}x \quad +\frac{1}{2}x$

$y = \frac{1}{2}x + 1$

Same line!

Infinitely many solutions
Consistent Dependent

2.) $x + y = 5$
$4 + y = -x$

$x + y = 5$

$-x \quad -x$

$y = -x + 5$

$4 + y = -x$

$-4 \quad -4$

$y = -x - 4$

Same slope, different y-int!
Parallel lines

No Solution
Inconsistent

3.) $y = 4x + 4$
$y - 3 = x$

$+3 \quad +3$

$y = x + 3$

*Intersecting Lines!

One Solution
Consistent Independent

4.) $2x - 3y = 6$
$y = \frac{2}{3}x$

$2x - 3y = 6$

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$

$y = \frac{2}{3}x - 2$

same slope, different y-int!
Parallel lines

No Solution
Inconsistent

Solve the system by substitution.

5.) $y = -2x + 4$
$2x + y = 4$

$2x - 2x + 4 = 4$
$4 = 4$ ✓
TRUE!

Infinitely many solutions
Consistent dependent

6.) $x + y = 2$
$2x + 2y = -6$

$x + y = 2$
$-x \quad -x$
$y = -x + 2$

$2x + 2(-x + 2) = -6$
$2x - 2x + 4 = -6$
$4 = -6$
False!

No Solution
Inconsistent

Solve the system by elimination.

7.) $-1(x + y = 3)$
$x + y = -1$
$-x - y = -3$

$0 = -4$
False

No Solution
Inconsistent

8.) $-9x - 3y = -18$
$3(3x + y = 6)$

$9x + 3y = 18$
$-9x - 3y = -18$

$0 = 0$
TRUE

Infinitely many solutions
Consistent dependent