

Accelerated Geometry
End of Year Algebra Review #1

Multi-Step Equations – use inverse operations to isolate the variable

Please note:

- When the variables cancel out on both sides of the equation, and you get a false statement, your answer is no solution. Use \emptyset .
- When the variables cancel out on both sides of the equation, and you get a true statement, your answer is all real numbers. Use TR.

Solve.

1. $8a = 2a + 30$

$-2a - 2a$

$\frac{6a}{6} = \frac{30}{6}$

$a = 5$

2. $\frac{2}{3}x - 7 = x$

$-\frac{2}{3}x - \frac{2}{3}x$

3. $-7 = \frac{1}{3}x$

$-21 = x$

3. $\frac{-6 - 4(y - 3)}{2} = y$

$-6 - 4y + 12 = 2y$

$6 - 4y = 2y$
 $+4y + 4y$

$6 = 6y$

$1 = y$

4. $2(g - 2) - 4 = 2(g - 3)$

$2g - 4 - 4 = 2g - 6$

$2g - 8 = 2g - 6$

$-8 \neq -6$

\emptyset

5. $3(30 + x) = 4(x + 19)$

$90 + 3x = 4x + 76$
 $-76 - 3x - 3x - 76$

$14 = x$

6. $\frac{1}{3}(12x - 21) = 4x - 7$

$4x - 7 = 4x - 7$ True!

all TR

How to Factor a...

Binomial	Trinomial	Polynomial of 4 terms
<ul style="list-style-type: none"> • GCF • Difference of Squares $a^2 - b^2 = (a + b)(a - b)$ • Sum or Difference of Cubes <ul style="list-style-type: none"> ○ Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ ○ Difference: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ 	<ul style="list-style-type: none"> • GCF • Perfect Square Trinomial $a^2 \pm 2ab + b^2 = (a \pm b)^2$ • Guess and Check 	<ul style="list-style-type: none"> • GCF • Factor by Grouping

* If not factorable, write prime.

Factor Completely.

7. $-t^3 - t^2 + 2t$ GCF!
 $-t(t^2 + t - 2)$
 $-t(t+2)(t-1)$

8. $25x^3 - 36y^2x$ GCF!
 $x(25x^2 - 36y^2)$ A.O.T.S.
 $x(5x-6y)(5x+6y)$

9. $64a^3 - 8$ GCF 1st!
 $8(8a^3 - 1)$ $a=2a$
 $b=1$
 $8(2a-1)(4a^2+2a+1)$

10. $r^4 - r^3 - 30r^2$ GCF!
 $r^2(r^2 - r - 30)$
 $r^2(r-6)(r+5)$

11. $20 - 60x + 45x^2$ GCF then P.S.T.
 $5(4 - 12x + 9x^2)$
 $5(2-3x)^2$

12. $12bc - 4bd + 15xc - 5xd$ Factor by grouping
 $4b(3c-d) + 5x(3c-d)$ Split in middle
 $(4b+5x)(3c-d)$

13. $8x^3 + 27$ $a=2x$
 $b=3$
 $(2x+3)(4x^2-6x+9)$

14. $x^2 - 24x + 144$ P.S.T.
 $(x-12)^2$

15. $4x^2 + 36y^2$ GCF
 $4(x^2 + 9y^2)$
 doesn't factor further!

16. $a^3 + 2a^2 - 5a - 10$ Split the middle
 $a^2(a+2) - 5(a+2)$
 $(a^2-5)(a+2)$

17. $x^3 - 27$ $a=x$
 $b=3$
 $(x-3)(x^2+3x+9)$

18. $10k^3 + 25k - 35k^2$ GCF!
 $5k(2k^2 - 7k + 5)$
 $5k(2k-5)(k-1)$

19. $125x^3 - 27$ $a=5x$
 $b=3$
 $(5x-3)(25x^2+15x+9)$

20. $5a^2z - 4a^2c + 15xz - 12xc$ Split middle
 $a^2(5z-4c) + 3x(5z-4c)$
 $(a^2+3x)(5z-4c)$

21. $y^3 - 8$ $a=y$
 $b=2$
 $(y-2)(y^2+2y+4)$

22. $-m^2 + mn + 2m - 2n$
 $-m(m-n) + 2(m-n)$
 $(-m+2)(m-n)$
 $(2-m)(m-n)$

23. $4x^2 + 6x + 9$
 Prime

24. $3x^3 + 3x^2 - 18x$ GCF!
 $3x(x^2 + x - 6)$
 $3x(x+3)(x-2)$

Guidelines to Solving/Finding Roots by Factoring

1. Transform the equation into standard form: $ax^2 + bx + c = 0$.
2. Factor the polynomial.
3. Set each factor equal to zero and solve.

*Leading coefficient should be positive.

**Check order of terms!

Solve.

25. $2x^2 = 10x - 12$

$2x^2 - 10x + 12 = 0$
 $2(x^2 - 5x + 6) = 0$
 $2(x-3)(x-2) = 0$
 $x-3=0$ $x-2=0$
 $x=3$ $x=2$

26. $3x^2 + 1 - 4x = 0$ *Watch order!*

$3x^2 - 4x + 1 = 0$
 $(3x-1)(x-1) = 0$
 $3x-1=0$ $x-1=0$
 $x = \frac{1}{3}$ $x = 1$

27. $3a^2 = 6a$

$3a^2 - 6a = 0$
 $3a(a-2) = 0$
 $3a = 0$ $a-2 = 0$
 $a = 0$ $a = 2$

28. $4(x-1)(x-3) = 0$ *already factored*

$x-1=0$ $x-3=0$
 $x=1$ $x=3$

29. $3p^2 + 17p = -10$

$3p^2 + 17p + 10 = 0$
 $(3p+2)(p+5) = 0$
 $3p+2=0$ $p+5=0$
 $p = -\frac{2}{3}$ $p = -5$

30. $x(2x-1)(2x+1) = 0$ *already factored*

$x=0$ $2x-1=0$ $2x+1=0$
 $2x=1$ $2x=-1$
 $x = \frac{1}{2}$ $x = -\frac{1}{2}$

31. $49x^2 - 225 = 0$

$(7x-15)(7x+15) = 0$
 $7x-15=0$ $7x+15=0$
 $x = \frac{15}{7}$ $x = -\frac{15}{7}$

32. $(z+1)(z-5) = 16$ *must = 0!*

$z^2 - 4z - 5 = 16$
 $-16 \quad -16$
 $z^2 - 4z - 21 = 0$
 $(z-7)(z+3) = 0$
 $z-7=0$ $z+3=0$
 $z = 7$ $z = -3$

33. $x(x-6) = 4(x-4)$

$x^2 - 6x = 4x - 16$
 $-4x + 16$
 $x^2 - 10x + 16 = 0$
 $(x-2)(x-8) = 0$
 $x-2=0$ $x-8=0$
 $x = 2$ $x = 8$

What if the quadratic equation is not factorable? Use Quadratic Formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- The equation must equal zero.
- Identify a, b, c values.
- Plug those values into the Quadratic Formula.
- Simplify to solve for x.

Solve.

34. $2x^2 + 6x + 3 = 0$

$a=2$
 $b=6$
 $c=3$
 $x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(3)}}{2(2)}$
 $x = \frac{-6 \pm \sqrt{36 - 24}}{4}$
 $x = \frac{-6 \pm \sqrt{12}}{4} = \frac{-3 \pm \sqrt{3}}{2}$

35. $2x^2 + 5x = 1$ *must = 0*

$2x^2 + 5x - 1 = 0$ $a=2$
 $b=5$
 $c=-1$
 $x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-1)}}{2(2)}$
 $x = \frac{-5 \pm \sqrt{25 + 8}}{4}$
 $x = \frac{-5 \pm \sqrt{33}}{4}$

36. $2m^2 = -4m + 2$

$2m^2 + 4m - 2 = 0$ *GCF!*
 $2(m^2 + 2m - 1) = 0$
 $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$ $a=1$
 $b=2$
 $c=-1$
 $x = \frac{-2 \pm \sqrt{4 + 4}}{2}$
 $x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$
 $x = -1 \pm \sqrt{2}$

Guidelines to solve fractional equations:

- Factor the denominator, if needed.
- State restrictions for the denominators.
 - Restrictions are stated when there is a variable in the denominator(s)
 - Set each factor equal to zero and solve for variable.
 - Then state that the variable cannot equal that number.
- Find the LCM of the denominators.
- Multiply each term by the LCM (this will cancel the denominators!).
- Solve the remaining equation.
- Check your solution with restriction(s)!
 - If a solution is a restriction, it cannot be an answer to equation

State restrictions and Solve.

$$37. \frac{y-2}{4y-8} = \frac{1}{4} \quad \boxed{y \neq 2}$$

$$y-2 = 1(y-2)$$

$$y-2 = y-2$$

$$\boxed{\text{all IR except } 2}$$

$$38. \frac{1}{y-3} = \frac{6}{y^2-9} \quad \boxed{y \neq \pm 3}$$

$$1(y+3) = 6$$

$$y+3 = 6$$

$$y-3 = 3$$

$$y = 3 \therefore \boxed{\emptyset}$$

$$39. \frac{1}{x-1} + \frac{x}{3} = \frac{x}{x-1} \quad \boxed{x \neq 1}$$

$$3 + x(x-1) = 3x$$

$$3 + x^2 - x = 3x$$

$$-3x - 3x$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\boxed{x=3} \quad x=1 \leftarrow \text{Restriction}$$

$$40. \frac{1}{x-4} + \frac{2}{x^2-16} = \frac{3}{x+4} \quad \boxed{x \neq \pm 4}$$

$$1(x+4) + 2 = 3(x-4)$$

$$x+4+2 = 3x-12$$

$$x+6 = 3x-12$$

$$-x+12 = 3x-12$$

$$18 = 2x$$

$$\boxed{9 = x}$$

$$41. \frac{2}{b^2-b} - \frac{2}{b-1} = 1 \quad \boxed{b \neq 0, 1}$$

$$2 - 2b = b(b-1)$$

$$2 - 2b = b^2 - b$$

$$-2 + 2b = b^2 - b$$

$$0 = b^2 + b - 2$$

$$0 = (b+2)(b-1)$$

$$\boxed{b = -2} \quad b = 1$$

↑
Restriction

$$42. \frac{x}{x-4} + \frac{1}{x-5} = \frac{-7}{x^2-9x+20} \quad \boxed{x \neq 5, 4}$$

$$x(x-5) + 1(x-4) = -7$$

$$x^2 - 5x + x - 4 = -7$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\boxed{x=3} \quad x=1$$