

Accelerated Geometry
End of Year Algebra Review #2

Guidelines to Simplifying Rational Expressions:

- Factor the numerator, if possible
- Factor the denominator, if possible
- Look for common factors, if any
- Cancel common factors

Express in simplest form. Assume no denominator equals zero.

$$1. \frac{x^2 + 6x + 8}{x^2 + 8x + 16} = \frac{\cancel{(x+4)}(x+2)}{\cancel{(x+4)}(x+4)}$$

$$\boxed{\frac{x+2}{x+4}}$$

$$2. \frac{a^2 + a - 2}{8 - 2a^2} = \frac{\cancel{(a+2)}(a-1)}{-2a^2 + 8}$$

$$= \frac{\cancel{(a+2)}(a-1)}{-2(a^2 - 4)}$$

$$= \frac{\cancel{(a+2)}(a-1)}{-2(a+2)(a-2)}$$

$$\boxed{-\frac{a-1}{2(a-2)}}$$

$$3. \frac{2n^2 + 5n + 3}{n^3 + n^2} = \frac{\cancel{(2n+3)}(n+1)}{n^2 \cancel{(n+1)}}$$

$$\boxed{\frac{2n+3}{n^2}}$$

$$4. \frac{3x^2 + 6x - 24}{3x^2 - 2x - 8} = \frac{3(x^2 + 2x - 8)}{(3x+4)(x-2)}$$

$$= \frac{3(x+4)\cancel{(x-2)}}{(3x+4)\cancel{(x-2)}}$$

$$\boxed{\frac{3(x+4)}{3x+4}}$$

Guidelines to Multiplying Rational Expressions:

- Factor the numerator and denominator of each fraction.
- Cancel common factors:
 - Up and Down and Diagonally
- Multiply numerator by numerator and denominator by denominator

Guidelines to Dividing Rational Expressions:

- Rewrite the problem into a multiplication problem.
 - First Fraction stays the same
 - Division sign changes to a multiplication sign
 - Take the reciprocal of the second fraction
- Factor the numerator and denominator of each fraction.
- Cancel common factors:
 - Up and Down and Diagonally
- Multiply numerator by numerator and denominator by denominator

Multiply or Divide. Assume no denominator equals zero.

$$5. \frac{2d-3e}{f^3} \cdot \frac{f}{3e-2d}$$

$$\frac{\cancel{2d-3e}}{\cancel{f^3} \cdot \cancel{f^2}} \cdot \frac{\cancel{f}}{-1(\cancel{2d-3e})}$$

$$\boxed{-\frac{1}{f^2}}$$

$$6. \frac{3ab}{9-b^2} \cdot \frac{2ab-6a}{6a^2b}$$

$$\frac{\cancel{3} \cancel{a} \cancel{b}}{3 \cancel{a} \cancel{b} (b+3)} \cdot \frac{\cancel{2} \cancel{a} \cancel{b} (b-3)}{\cancel{6} \cancel{a} \cancel{a} \cancel{b}}$$

$$\boxed{-\frac{1}{b+3}}$$

$$7. \frac{y}{2x^3} \div \left(\frac{y}{2x}\right)^2$$

$$\frac{\cancel{y}}{\cancel{2} \cancel{x} \cancel{x} \cancel{x}} \cdot \frac{\cancel{2} \cancel{x} \cancel{x}}{\cancel{y} \cancel{y}} = \boxed{\frac{2}{xy}}$$

$$8. \frac{a^2-a-12}{a^2-4a} \div \frac{a^2+2a-3}{2a^2}$$

$$\frac{a^2-a-12}{a^2-4a} \cdot \frac{2a^2}{a^2+2a-3}$$

$$\frac{(a-4)(a+3)}{\cancel{a} \cancel{a} (a-4)} \cdot \frac{2a^2}{(a+3)(a-1)}$$

$$\boxed{\frac{2a}{a-1}}$$

$$9. \frac{ac+ad}{ac-ad} \div \frac{5c+5d}{c^2-d^2}$$

$$\frac{ac+ad}{ac-ad} \cdot \frac{c^2-d^2}{5c+5d}$$

$$\frac{a(c+d)}{\cancel{a} \cancel{c} \cancel{a}} \cdot \frac{(c+d)(c+d)}{5(c+d)}$$

$$\boxed{\frac{c+d}{5}}$$

$$10. \frac{d^2+9d+18}{2d^2+4d} \div \frac{d^2+d-6}{d^2-4}$$

$$\frac{d^2+9d+18}{2d^2+4d} \cdot \frac{d^2-4}{d^2+d-6}$$

$$\frac{(d+3)(d+6)}{2d(d+2)} \cdot \frac{(d-2)(d+2)}{(d+3)(d-2)}$$

$$\boxed{\frac{d+6}{2d}}$$

Guidelines to Adding and Subtracting Expressions:

1. Check to see if the fractions have the same denominator.
2. If not, find the least common multiple of the denominators
3. Rewrite the problem using the LCD.
4. Add/subtract the numerators and keep the same denominator.
5. Simplify your answer by looking for common factors.

Recall: You may have to factor to simplify.

Add or Subtract. Assume no denominator equals zero.

$$11. \frac{4a^2}{a^2+a} - \frac{3a^2+1}{a^2+a}$$

$$\frac{4a^2-3a^2-1}{a^2+a} = \frac{a^2-1}{a^2+a}$$

$$= \frac{(a-1)(a+1)}{a(a+1)} = \boxed{\frac{a-1}{a}}$$

$$12. \frac{2}{x-1} + \frac{9}{4x-4}$$

LCD = $4(x-1)$

$$\frac{8}{4(x-1)} + \frac{9}{4(x-1)} = \boxed{\frac{17}{4(x-1)}}$$

$$13. \frac{6a-10}{a^2-a-12} + \frac{4}{a+3}$$

LCD = $(a+3)(a-4)$

$$\frac{6a-10}{(a+3)(a-4)} + \frac{-4a+16}{(a+3)(a-4)}$$

$$\frac{2a+6}{(a+3)(a-4)} = \frac{2(a+3)}{(a+3)(a-4)} = \boxed{\frac{2}{a-4}}$$

$$14. \frac{3a}{a-2b} + \frac{6b}{2b-a}$$

$$\frac{3a}{a-2b} - \frac{6b}{a-2b} = \frac{3a-6b}{a-2b}$$

$$= \frac{3(a-2b)}{a-2b} = \boxed{3}$$

$$15. \frac{2a(a+3)}{6a^2-12a} + \frac{5(a-2)}{12a^2}$$

LCD = $12a^2(a-2)$

$$\frac{2a^2+6a}{12a^2(a-2)} + \frac{5a-10}{12a^2(a-2)} = \boxed{\frac{2a^2+11a-10}{12a^2(a-2)}}$$

$$16. \frac{3}{1} - \frac{2x-2}{x^2-1}$$

LCD = x^2-1

$$\frac{3x^2-3}{x^2-1} - \frac{2x-2}{x^2-1}$$

$$\frac{3x^2-3-2x+2}{x^2-1} = \frac{3x^2-2x-1}{x^2-1}$$

$$\frac{(3x+1)(x-1)}{(x-1)(x+1)} = \boxed{\frac{3x+1}{x+1}}$$

Simplify. Assume no denominator equals zero.

$$17. \frac{x}{x+2} - \frac{4}{x-2} - 1$$

LCD = $(x+2)(x-2)$

$$\frac{x^2-2x}{(x+2)(x-2)} - \frac{4x+8}{(x+2)(x-2)} - \frac{x^2-4}{(x+2)(x-2)}$$

$$\frac{x^2-2x-4x-8-x^2+4}{(x+2)(x-2)}$$

$$\boxed{\frac{-6x-4}{(x+2)(x-2)}} = \frac{-2(3x+2)}{(x+2)(x-2)}$$

Nothing cancels!

$$18. \frac{z-5z}{z+5} + \frac{5z}{z-5}$$

← Simplify numerators

← Simplify denominators

$$\frac{z(z+5)}{1} - \frac{5z}{z+5}$$

$$\frac{z^2+5z}{z+5} - \frac{5z}{z+5} = \frac{z^2+5z-5z}{z+5} = \frac{z^2}{z+5}$$

$$\frac{z(z-5)}{1} + \frac{5z}{z-5}$$

$$\frac{z^2-5z}{z-5} + \frac{5z}{z-5} = \frac{z^2-5z+5z}{z-5} = \frac{z^2}{z-5}$$

$$\frac{z^2}{z+5} \div \frac{z^2}{z-5} = \frac{z^2}{z+5} \cdot \frac{z-5}{z^2} = \frac{z-5}{z+5}$$

Final Answer

Polynomial Long Division

- Polynomials must be in descending order.
- If its missing a term, use 0 as its coefficient so that every term is expressed.
- If there's a remainder, be sure to express as $\pm \frac{\text{remainder}}{\text{divisor}}$ as part of the quotient.

Use polynomial long division to find the quotient.

19. $2x-4 \overline{) 2x^3 - 10x^2 + 14x - 4}$

Handwritten solution:

$$\begin{array}{r} \boxed{x^2 - 3x + 1} \\ 2x-4 \overline{) 2x^3 - 10x^2 + 14x - 4} \\ \underline{-2x^3 + 8x^2} \\ -8x^2 + 14x \\ \underline{+ 6x^2 - 12x} \\ -2x + 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

20. $\frac{45 - 13n + n^2}{n-5}$

Handwritten solution:

Watch order!

$$\begin{array}{r} \boxed{n - 8 + \frac{5}{n-5}} \\ n-5 \overline{) n^2 - 13n + 45} \\ \underline{-n^2 + 5n} \\ -8n + 45 \\ \underline{+ 8n - 40} \\ 5 \end{array}$$

21. $\frac{a^3 + 27}{a+3}$

Handwritten solution:

missing terms!

$$\begin{array}{r} \boxed{a^2 - 3a + 9} \\ a+3 \overline{) a^3 + 0a^2 + 0a + 27} \\ \underline{-a^3 + 3a^2} \\ -3a^2 + 0a \\ \underline{+ 3a^2 - 9a} \\ -9a + 27 \\ \underline{+ 9a - 27} \\ 0 \end{array}$$

22. $(6x^3 + 11x^2 - 14x - 10) \div (2x + 5)$

Handwritten solution:

$$\begin{array}{r} \boxed{3x^2 - 2x - 2} \\ 2x+5 \overline{) 6x^3 + 11x^2 - 14x - 10} \\ \underline{-6x^3 + 15x^2} \\ -4x^2 - 14x \\ \underline{+ 4x^2 + 10x} \\ -4x - 10 \\ \underline{+ 4x + 10} \\ 0 \end{array}$$