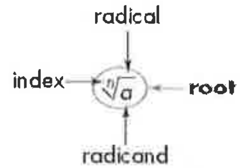
Name Answer Key Date ______________ Period ________

Accelerated Geometry
End of Year Algebra Review #4

Root: The *nth* root of a given number is a specific number that when it is used as a factor n times, equals the given number.

Radicand: the expression under the radical sign.

Index: the degree of the root.



Finding Roots

- Look at the index to see how many factors you need.
- Use factor trees, if needed, to help you determine factors.

Recall: $\sqrt{x} = x^{\frac{1}{2}}$ $\quad \sqrt[3]{x} = x^{\frac{1}{3}}$ $\quad \sqrt[4]{x} = x^{\frac{1}{4}}$ $\quad \sqrt[5]{x} = x^{\frac{1}{5}}$

Consequently, we can say $\sqrt[n]{x} = x^{\frac{1}{n}}$. Moreover, we can say $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ and $\left(\sqrt[n]{x}\right)^m = x^{\frac{m}{n}}$

Simplify.

1. $\sqrt[5]{243} = \boxed{3}$

   3, 81; 9, 9; 3 3 3 3

2. $\sqrt[3]{-8x^{21}y^2} = \boxed{-2x^7\sqrt[3]{y^2}}$

3. $\sqrt[4]{625} = \boxed{5}$

   25, 25; 5 5 5 5

4. $\sqrt{\dfrac{121}{289}} = \boxed{\dfrac{11}{17}}$

5. $\pm\sqrt[6]{1} = \boxed{\pm 1}$

6. $\sqrt[3]{(x-9)^3} = \boxed{|x-9|}$

7. $81^{-0.75}$ ← convert to fraction!

   $81^{-\frac{3}{4}} = \dfrac{1}{81^{\frac{3}{4}}} = \dfrac{1}{\sqrt[4]{81}^3} = \dfrac{1}{3^3} \boxed{\dfrac{1}{27}}$

8. $(243)^{\frac{2}{5}} = \sqrt[5]{243}^2$

   $= 3^2 = \boxed{9}$

9. $-64^{\frac{2}{3}} = \sqrt[3]{-64}^2$

   $(-4)^2 = \boxed{16}$

Write in exponential form.

10. $\sqrt{x^3}$ $\quad \boxed{x^{\frac{3}{2}}}$

11. $\sqrt[3]{2y^2}$ $\quad \boxed{2^{\frac{1}{3}}y^{\frac{2}{3}}}$

12. $(\sqrt[4]{b})^3$ $\quad \boxed{b^{\frac{3}{4}}}$

13. $\sqrt{-6}$ $\quad \boxed{(-6)^{\frac{1}{2}}}$

14. $\sqrt[4]{(5ab)^3}$ $\quad \boxed{(5ab)^{\frac{3}{4}}}$

15. $\sqrt[5]{\dfrac{3x^2}{z^{10}}}$ $\quad \boxed{\dfrac{3^{\frac{1}{5}}x^{\frac{2}{5}}}{z^2}}$

Properties of Radicals

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$$ *Product Property: index must be the same

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$$ *Quotient Property: index must be the same

Take note: If the index is not the same, you must apply your exponent rules!

Sums/Differences of Radicals

When Adding and Subtracting radicals: simplify each term and look to see if you can combine like terms.

- Remember the radical stays the same when you add or subtract the coefficients of the radicals.

Binomials containing Radicals: FOIL with radical operations

Conjugate: expressions that differ only in signs of the 2nd terms

Example: $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$

Simplify each expression.

16. $6\sqrt{18}+3\sqrt{50}$

$6\cdot 3\sqrt{2}+3\cdot 5\sqrt{2}$

$18\sqrt{2}+15\sqrt{2}=\boxed{33\sqrt{2}}$

17. $3\sqrt[3]{81}-2\sqrt[3]{54}-\sqrt[3]{2}$

$3\cdot 3\sqrt[3]{3}-2\cdot 3\sqrt[3]{2}-\sqrt[3]{2}$

$9\sqrt[3]{3}-6\sqrt[3]{2}-\sqrt[3]{2}$

$\boxed{9\sqrt[3]{3}-7\sqrt[3]{2}}$

18. $3\sqrt{2}\left(5\sqrt{8}-2\sqrt{2}\right)$ distribute!

$15\sqrt{16}-6\cdot 2$

$15\cdot 4-12$

$60-12$

$\boxed{48}$

19. $\left(5+2\sqrt{5}\right)\left(7+4\sqrt{5}\right)$ FOIL

$35+20\sqrt{5}+14\sqrt{5}+8\cdot 5$

$35+34\sqrt{5}+40$

$\boxed{75+34\sqrt{5}}$

20. $\left(2\sqrt{5}-\sqrt{2}\right)^2$ ← write twice & FOIL

$(2\sqrt{5}-\sqrt{2})(2\sqrt{5}-\sqrt{2})$

$4\cdot 5-2\sqrt{10}-2\sqrt{10}+2$

$20-4\sqrt{10}+2$

$\boxed{22-4\sqrt{10}}$

21. FOIL!

$\frac{(5+\sqrt{3})}{(2-\sqrt{3})}\cdot\frac{(2+\sqrt{3})}{(2+\sqrt{3})}=\frac{10+5\sqrt{3}+2\sqrt{3}+3}{4-3}$

Rationalize

Multiply by conjugate

$\frac{13+7\sqrt{3}}{1}=\boxed{13+7\sqrt{3}}$

Simplify each expression.

22. $\sqrt{\frac{50m^2n^6}{8m^4n^5}}$ Simplify radicand 1st!

$\sqrt{\frac{25n}{4m^2}} = \boxed{\frac{5\sqrt{n}}{2|m|}}$

23. $-\frac{9x^3}{\sqrt{18x^5}} = -\frac{9x^3}{3x^2\sqrt{2x}} = \frac{3x}{\sqrt{2x}}\cdot\frac{\sqrt{2x}}{\sqrt{2x}} = \frac{3\cancel{x}\sqrt{2x}}{2\cancel{x}}$

Simplify

Simplify 1st!

Rationalize!

$\boxed{\frac{3\sqrt{2x}}{2}}$

24. $\frac{4+\sqrt{27}}{2-3\sqrt{27}} = \frac{(4+3\sqrt{3})}{(2-9\sqrt{3})}\cdot\frac{(2+9\sqrt{3})}{(2+9\sqrt{3})}$

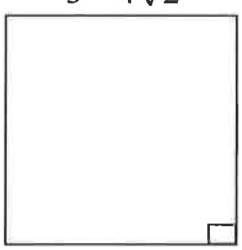
$\frac{8+36\sqrt{3}+6\sqrt{3}+27\cdot 3}{4-81\cdot 3}$

$= \frac{8+42\sqrt{3}+81}{4-243} = \frac{89+42\sqrt{3}}{-239} = \boxed{\frac{-89-42\sqrt{3}}{239}}$

25. $\frac{4+\sqrt{6}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}+\sqrt{18}}{3} = \boxed{\frac{4\sqrt{3}+3\sqrt{2}}{3}}$

Find the area and perimeter of the rectangle below.

26. $3-4\sqrt{2}$



$5-6\sqrt{2}$

$A = lw$

$A = (3-4\sqrt{2})(5-6\sqrt{2})$

$A = 15-18\sqrt{2}-20\sqrt{2}+24\cdot 2$

$A = 15-38\sqrt{2}+48$

$\boxed{A = 63-38\sqrt{2}\ un^2}$

$P = 2l+2w$

$P = 2(3-4\sqrt{2})+2(5-6\sqrt{2})$

$P = 6-8\sqrt{2}+10-12\sqrt{2}$

$\boxed{P = 16-20\sqrt{2}\ un}$

Square Root of a Negative Real Number

- $i = \sqrt{-1}$
- *For any positive number a*
  $\sqrt{-a} = \sqrt{a\cdot -1} = \sqrt{-1}\cdot\sqrt{a} = i\sqrt{a}$
- $i^2 = -1$
- Complex Number $a \pm bi$ ← imaginary
  ↑ Real

Simplify.

27. $\sqrt{-128} = 2\cdot 2\cdot 2i\sqrt{2}$

4 32

$\boxed{8i\sqrt{2}}$

28. $\sqrt{-5}\bullet\sqrt{-60}$

$\sqrt{300}$

$\boxed{10\sqrt{3}}$

29. $(2i\sqrt{3})^2$ Product to a power

$2^2i^2(\sqrt{3})^2$

$4(-1)\cdot 3$

$\boxed{-12}$

Adding, Subtracting, Multiplying and Dividing Complex Numbers

30. $(8 + 2i) + (3 - 4i)$

$\boxed{11 - 2i}$

31. $(3 - 5i) - (-1 + 7i)$ distribute

$3 - 5i + 1 - 7i$

$\boxed{4 - 12i}$

32. $\frac{4+3i}{9i} \cdot \frac{9i}{9i} = \frac{36i + 27i^2}{81i^2}$ $* i^2 = -1!$

Rationalize

$\frac{36i + 27(-1)}{81(-1)} = \frac{36i - 27}{-81}$

$= \frac{27 - 36i}{81} = \boxed{\frac{3 - 4i}{9}}$

33. $(-1 + 2i)(3 + 10i)$ FOIL

$-3 - 10i + 6i + 20i^2$ $* i^2 = -1!$

$-3 - 4i + 20(-1)$

$-3 - 4i - 20$

$\boxed{-23 - 4i}$

34. $(4 - i)^2$ ← write twice & FOIL!

$(4 - i)(4 - i)$

$16 - 4i - 4i + i^2$ $* i^2 = -1!$

$16 - 8i + (-1)$

$\boxed{15 - 8i}$

35. $\frac{7}{5-2i} \cdot \frac{5+2i}{5+2i}$

$\frac{35 + 14i}{25 - 4i^2} = \frac{35 + 14i}{25 - 4(-1)}$

$= \frac{35 + 14i}{25 + 4} = \boxed{\frac{35 + 14i}{29}}$