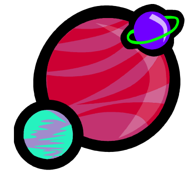


Kepler's Laws

1st Law: The planets move about the sun in ELLIPTICAL orbits, with the sun at one focus of the ellipse.

2nd Law: The straight line joining the sun and a given planet sweeps equal areas in equal amounts of time.

→ Can be remembered as the "SLINGSHOT EFFECT"

3rd Law: The square of the period of revolution of a planet <u>ABOUT THE SUN</u> is proportional to the cube of its mean distance from the sun.

→ Stated in equation form as $K = R^3 / T^2$

Defining the variables:

T = Orbital Period (time needed to complete 1 full orbit)

R = Mean (or average) Orbital Radius

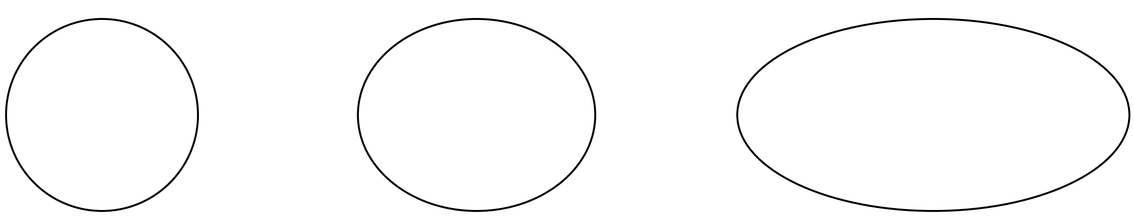
r = the radius of the planet itself

Some key things to remember/know about Kepler's Laws

<u>1st Law:</u>

- ✓ Circles have centers. Ellipses are like flattened circles, that don't have a center, but rather have two foci.
- ✓ ***<u>Eccentricity</u>*** may be interpreted as a measure of how much an orbit's shape deviates from a circle.

For a circle, $e = 0$
For an ellipse, $0 < e < 1$ (the lower the e value, the more circular the orbit)



- ✓ $e = 0.017$ for Earth's orbit, $e = 0.093$ for Mars' orbit, $e = .252$ for Pluto's orbit

2^{nd} Law:

- ✓ Planets move ___fast___ when they are on the side of their elliptical orbit that is closest to the sun.

- ✓ Between March 21 and September 21, there are three days more than between September 21 and March 21. These two dates are the spring and fall equinoxes, when the days and nights are of equal length. Between the equinoxes, the Earth moves 180° around its orbit with respect to the sun. Using Kepler's 2^{nd} Law, explain clearly how you can determine the part of the year during which the Earth is closer to the sun.

  Even though it would make sense that the earth is closer to the sun when the temperature on earth is hotter (in the summer), this is actually not true. The earth moves faster when it is on the part of it elliptical orbit that is closest to the sun. This would mean that it will spend less days close to the sun. Since the winter has less days than the summer, the Earth must in fact be closer to the sun in the winter!

3^{rd} Law:

- ✓ All planets that orbit the sun have the same Kepler Constant (which equals ___3.35×10^{18}___)
- ✓ All "things" (little) that orbit the same "THING" (BIG) have the same Kepler constant.
- ✓ The orbital period of the earth about the sun is approximately ___365 days___.
- ✓ The orbital period of the moon about the earth is approximately ___27 days___.
- ✓ When using Kepler's 3^{rd} Law, make sure to use units of METERS and SECONDS.

- ✓ The units of K are ___m^3/s^2___.

- ✓ When using your calculator with BIG numbers that involve exponents, make sure to utilize parenthesis properly, making sure to pay attention to the ORDER OF OPERATIONS (remember PEMDAS)

- ✓ If you solve Kepler's 3^{rd} Law for R, it will involve a cube root. There are two ways to do this on your calculator.

"Kepler's Laws" Worksheet (SOLUTIONS)

#1

$3.35E18\frac{m^3}{s^2}$; $3.35E18\frac{m^3}{s^2}$; $3.35E18\frac{m^3}{s^2}$; $3.35E18\frac{m^3}{s^2}$; $3.35E18\frac{m^3}{s^2}$; $3.35E18\frac{m^3}{s^2}$; $3.34E18\frac{m^3}{s^2}$; $3.37E18\frac{m^3}{s^2}$; $3.36E18\frac{m^3}{s^2}$; $1.01E13\frac{m^3}{s^2}$

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#2

$K = \frac{R^3}{T^2} = 3.35E18\frac{m^3}{s^2}$ (for any planet that orbits the sun, as seen in the table on the last page of this WS)

$$T = \sqrt{\frac{R^3}{K}} = \sqrt{\frac{(2E11m)^3}{3.35E18\frac{m^3}{s^2}}} = 4.9\times10^7 \text{ sec}$$

~~~~~~

#3

$K = \frac{R^3}{T^2}$ (for any planet orbiting the sun) $= 3.35E18\frac{m^3}{s^2}$

$$T_{sp} = 2T_e = 2 * 3.16E7\text{ sec} = 6.32E7\text{ sec}$$

$$R_{sp}{}^3 = KT^2 \rightarrow R_{sp} = \sqrt[3]{KT^2} = \sqrt[3]{(3.35E18)(6.32E7)^2} = 2.374E11m$$

Since $R_e = 1.49E11m$, then $\frac{R_{sp}}{R_e} = \frac{2.374E11m}{1.49E11m} = 1.6$

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#4

Using the moon,

$R = 384.4E3\text{ km} = 384.4E6\text{ m}$

$T = 27.322\text{ days} = 2{,}360{,}620.8\text{ sec}$

$$K = \frac{R^3}{T^2} = \frac{(384.4E6m)^3}{(2{,}360{,}620.8\text{ sec})^2} = 1.01\times10^{13}\frac{m^3}{s^2}$$

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#5

Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune

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#6

Neptune has a larger R value. However, since its K value is the same as that of Saturn, its T value must also be larger in order to compensate. Therefore, Neptune must have a larger orbital period, and thus takes longer to orbit the sun.

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#7

Using the same procedure as in #3 above,

Earth: 1.01E13 m^3/s^2
Mars: 1.09 E12 m^3/s^2
Jupiter: 3.21E15 m^3/s^2
Saturn: 9.64E14 m^3/s^2
Uranus: 1.45E14 m^3/s^2
Neptune: 1.74E14 m^3/s^2
Pluto: 2.51E10 m^3/s^2

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#8

$K = \frac{R^3}{T^2} = 3.35E18\frac{m^3}{s^2}$ (for any planet that orbits the sun, as seen in the table on the last page of this WS)

$$T = \sqrt{\frac{R^3}{K}} = \sqrt{\frac{(4.8E11m)^3}{3.35E18\frac{m^3}{s^2}}} = 1.82\times10^8 \text{ sec}$$

~~~~~~

#9

Since $K = \frac{R^3}{T^2} = 1.01E13\frac{m^3}{s^2}$ for anything that orbits the earth (from problem #5 above),

$$K = \frac{R^3}{T^2} \quad\rightarrow\quad T = \sqrt{\frac{R^3}{K}} = \sqrt{\frac{(2r_e)^3}{K}} = \sqrt{\frac{(2\cdot 6.38E6m)^3}{1.01E13\frac{m^3}{s^2}}} = 1.43\times10^4 \text{ sec}$$

~~~~~~

#10

$R_P = ???;\quad R_D = 2.3E4km = 2.3E7m$

$T_P = 7h39\min = 27{,}540\sec \quad T_D = 30h18\min = 109{,}080\sec$

Since $K = \frac{R^3}{T^2} = \text{constant}$,

$$\frac{R_D{}^3}{T_D{}^2} = \frac{R_P{}^3}{T_P{}^2} \quad\rightarrow\quad R_P{}^3 = \frac{T_P{}^2 R_D{}^3}{T_D{}^2} \quad\rightarrow\quad R_P = \sqrt[3]{\frac{T_P{}^2 R_D{}^3}{T_D{}^2}} = \sqrt[3]{\frac{(27{,}540\sec)^2(2.3E7m)^3}{(109{,}080\sec)^2}} = 9.2\times10^6 m$$

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#11

Since $K = \frac{R^3}{T^2} = 1.09E12\frac{m^3}{s^2}$ for anything that orbits Mars (from problem #5 above),

$$K = \frac{R^3}{T^2} \quad\rightarrow\quad T = \sqrt{\frac{R^3}{K}} = \sqrt{\frac{(3.43E6+100{,}000m)^3}{1.09E12}} = 6352.6\sec = 105.9\min = 1.76h$$

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#12

Since $K = \frac{R^3}{T^2} = 1.01E13\frac{m^3}{s^2}$ for anything that orbits the Earth (from problem #5 above),

Since the earth rotates once every 8.61E4sec (from the table above), T = 8.61E4sec

$$K = \frac{R^3}{T^2} \quad\rightarrow\quad R = \sqrt[3]{T^2K} = \sqrt[3]{(8.61E4\sec)^2(1.01E13\frac{m^3}{s^2})} = 4.21E7m$$

Therefore, the radius measured from the center of the earth is 4.2×10^7m.

The distance from the earth's surface is $4.21\times10^7\text{m} - r_e = 4.21E7\text{m} - 6.38E6\text{m} = 3.6\times10^7\text{m}$.