In an ELASTIC collision, energy is conserved \((KE_{\text{before}} = KE_{\text{after}} \text{ or } K_i = K_f)\).

In an INELASTIC collision, energy is NOT conserved. \((K_i > K_f)\).

Example: A 1 kg block which is sliding at 10 m/s across a frictionless surface suddenly collides with a stationary 2 kg block. They stick together and they move towards an inclined plane of angle 37°. How far up the incline will they move? Was the collision elastic or inelastic?

Example: A 10 kg pendulum bob is hanging at rest at the end of a 4 meter long rope. A 500 gram ball is thrown horizontally at the pendulum bob with a speed of 5.0000 m/s. It strikes the pendulum bob, and they bounce straight backward at a speed of 4.5238 m/s. What angle will the pendulum bob swing through? Was the collision elastic or inelastic?
For ELASTIC, HEAD-ON, 1D Collisions

\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

**Conservation of Momentum:**

*If we know all but one velocity (or mass), we can find the missing velocity (or mass) \( \ominus \).*

\[ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]

**Conservation of Energy:**

Put the two equations together (carefully and with many steps \( \ominus \)) and you get the following equations.

\[ v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{2i} \quad v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} - \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{2i} \]

*With these equations, we can find BOTH velocities of the balls after the collision by just using the initial velocity of the balls and the masses of the balls! \( \ominus \, \ominus \, \ominus \, \ominus \, \ominus \)*

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**Special Case:** If \( v_{2i} = 0 \), then what happens when .....

- a) \( m_1 > m_2 \)
- b) \( m_1 < m_2 \)
- c) \( m_1 \gg m_2 \)
- d) \( m_1 \ll m_2 \)
- e) \( m_1 = m_2 \)
Collision HW Problems:

1. A cue ball moving to the right strikes the 8-ball (of equal mass, and at rest) in an elastic, head-on collision. If the cue ball’s velocity was initially 5 m/s, find the velocity of both balls after the collision.

2. A 4 kg red box is slid across a frictionless floor with a velocity of 8 m/s. If it strikes a stationary 3 kg blue box in an elastic head-on collision, find the velocity of both box’s after the collision.

3. A 1 kg block is sitting on a frictionless table. It is struck by a 5 g bullet that is traveling at 250 m/s. The bullet lodges itself in the block, which then begins to slide. The bullet-block then collides (elastically, and head-on, of course) with a 50 kg box that is at rest on the table. Find the post-collision velocities of the bullet-block and the box.

4. A 4000 kg truck traveling at 10 m/s slams into a 20 kg grocery cart at rest in a parking lot. Find the speed of both objects after this elastic, head-on collision.

5. A 4000 kg truck traveling at 10 m/s slams into a 20 kg grocery cart moving towards it at 5 m/s. Find the speed of both objects after this elastic, head-on collision.

6. A 500 kg go-cart moving at 20 m/s rear ends a 300 kg scooter moving at 10 m/s. After the collision, the go-cart bounces backward at a speed 10 m/s, while the scooter is sped up to an unknown speed. Find the scooter’s speed. Is the collision elastic or inelastic? Explain why this collision is IMPOSSIBLE.

Collision / Pendulum / Energy-Related HW Problems:

7. A ball on a 50 cm string is hung from the ceiling. If the ball is pulled back through an angle of 40° (with the vertical) and then released, find the velocity of the ball when it reaches the bottom of its swing. (Hint: Use the conservation of energy)

8. The ball from problem #4 breaks at the exact moment that it reaches its maximum velocity, thus becoming a projectile. Find the horizontal distance that it will travel in the air (after the string snaps) before hitting the ground 2.5 m below.

9. Collision & Pendulum Combined 🎳. The 1 kg ball is hung from a string and hung from a surface above a frictionless table. A 5 kg block is slid across the table with a constant velocity of 1 m/s. If it strikes the ball in a perfectly elastic, head-on collision, find the angle that the pendulum will swing through before coming back down.

Answers:

1) Cue: 0 m/s, 8-ball: 5 m/s →
2) Red: 1.143 m/s, Blue: 9.143 m/s
3) \(v_{\text{block/bullet},i} = 1.244 \text{ m/s}, \ v_{\text{block/bullet},f} = -1.195 \text{ m/s}, \ v_{\text{box},f} = .049 \text{ m/s}\)
4) Truck: 9.9 m/s, S-Cart: 19.9 m/s
5) Truck: 9.85 m/s, S-Cart: 24.85 m/s
6) 60 m/s, Inelastic, ________
7) 1.514 m/s
8) \(\Delta t = .714 \text{ sec}, \ \Delta x = 1.08 \text{ m}\)
9) \(v_{\text{bullet}} = 1.67 \text{ m/s}, \ h = .142 \text{ m}, \ \theta = 31^\circ\)